# Spare Parts Management

## for Large-scale Fleet Scenarios

## ABSTRACT

In managing spare parts for large-scale fleet scenarios, one needs to optimally allocate spares across multi-echelons of maintenance agencies. This paper discusses an in-house developed simulation model (PIPER) for the Army to solve problems such as the evaluation of maintenance support concept, the impact of combat damage during wartime and workshop loading. PIPER is validated successfully against commercially available tools with good agreement between the models. It uses the combined technique of analytical marginal analysis while heuristics are employed for optimising spares and maintenance resources. The article also discusses a demand forecasting model that addresses the different equipment reliability when they are operated in different environments. This model is based on a multi-population mixture of Weibull failure distributions.

**See Chuen Teck**

# Spare Parts Management
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## INTRODUCTION

It is well known that spare parts management for large-scale fleet scenarios is a complex problem. In particular, one needs to optimally allocate spare parts across multi-echelons of maintenance agencies. The Pipeline Simulator (PIPER) is a Monte Carlo simulation model developed by DSTA to manage spare parts for the Army. The model solves problems such as the evaluation of maintenance support concept, the impact of combat damage and workshop loading. This model provides full access to the source code for customisation and integration with other models or Management Information Systems. It is also designed to be scalable whereby models and new functionalities are created via the addition of 'building blocks'. It analyses multiple combat units, quantifies the effect of sharing spares and men, handles war scenario (time varying utilisation rate, combat damage and attrition, base open and close) and explicitly models repair manpower, heavy transporter and periodic re-supply. A combinatorial technique of analytical marginal analysis and heuristics is employed for optimising spares and maintenance resources in PIPER.

The first step to good management of spare parts is to forecast demands realistically from real life data. However, in many operational fleets, differences in field operating conditions, usage intensity and product inception time often result in varying age and conditions of the equipment. While the case of homogenous equipment failure characteristic has been popular, it is inappropriate in many military systems where the deployment of the fleet is dispersed over a multiplicity of operating environments. To improve the accuracy of PIPER in optimising spares and maintenance resources, a demand forecasting system for spare parts management in large-scale fleet scenarios with substantial variability in equipment reliability is being developed. Our model is based on a multi-population mixture of Weibull failure distributions.

## MODEL

The PIPER model is built using Extend (developed by Imagine That Inc.), a simulation tool widely used by academics and the simulation industry. The PIPER model shown in Figure 1 consists of four echelons of repair

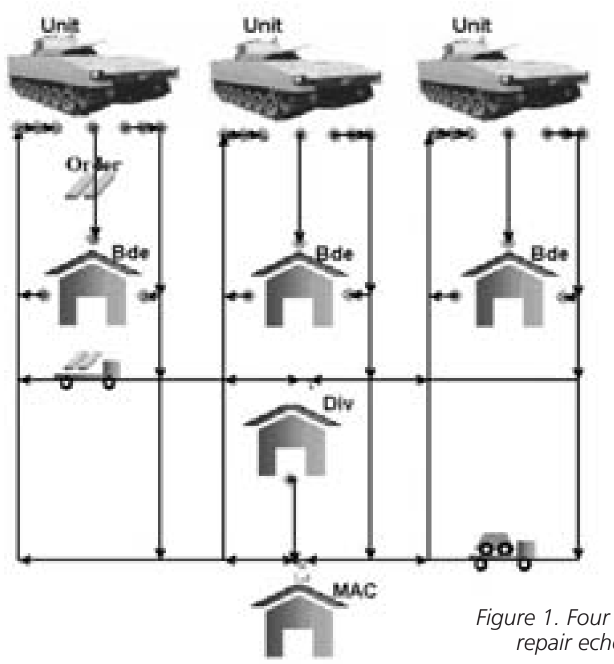



*Figure 1. Four levels of repair echelon*

agencies. Systems can be deployed at any of the four repair echelons. System repair is carried out at the second, third and fourth repair echelons. All four echelons hold a suite of maintenance resources i.e. test equipment and technicians to service the repair jobs. The maintenance resources follow a user-defined operation schedule (the base open and close timing). The quantity of maintenance resources is allowed to change over time.

The transport time and milk-run frequency among the various repair agencies could be defined to take special values and to override the default parameters. This may be used to represent certain Line Replacement Units (LRUs) transported by special mechanisms e.g. helicopter lift, pseudo stores or repair echelons. The milk-run frequency may be variable over the timeline and a frequency of zero milk-run can be used to represent a temporary stoppage of supply i.e. enemy action or truck getting 'lost'.

The model is synthesised from building blocks present in the PIPER libraries as shown in Figures 2 and 3.

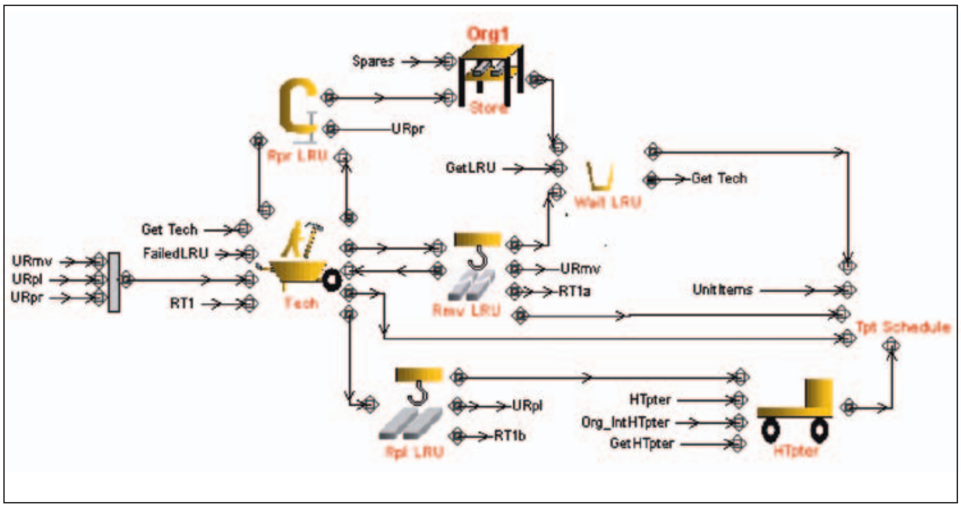



*Figure 2. Unit level*

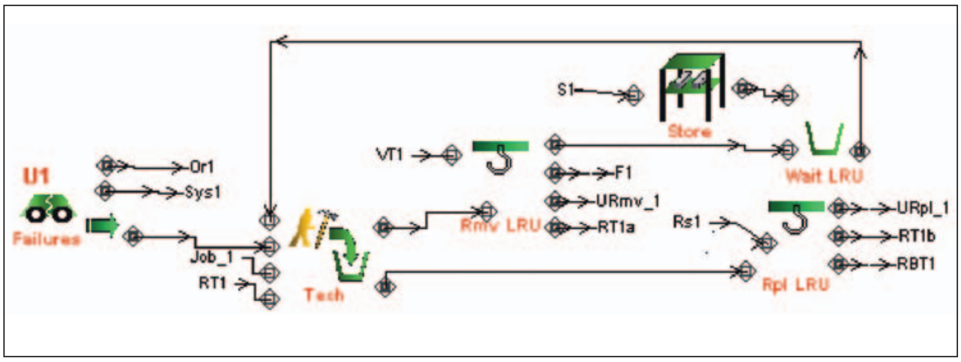



*Figure 3. Second repair echelon*

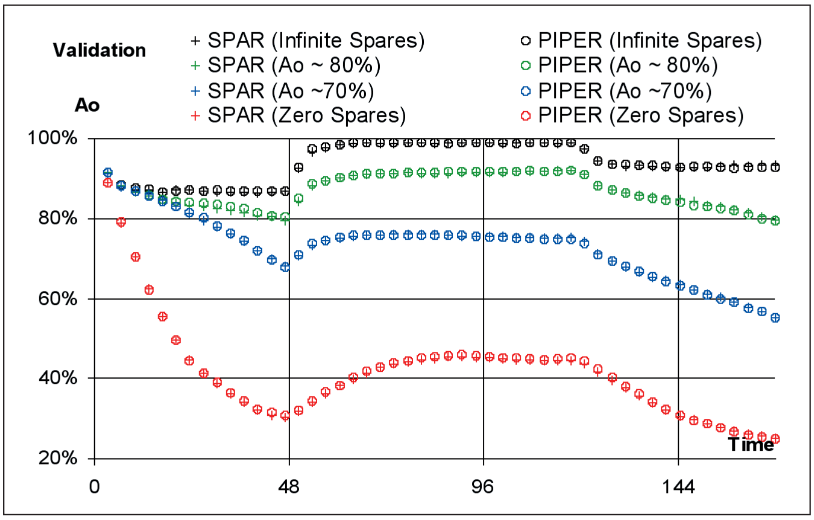



*Figure 4. Validation results from two Army case studies*

Validation against other commercial tools such as SPAR (a tool developed by Clockwork Solutions) shows good agreement between the two models. Shown in Figure 4 is the result of validation from two Army case studies. It should be highlighted that the validation focused on the simulation aspect of the model. Validation on the optimisation algorithm was not addressed in this portion.

## OPTIMISATION HEURISTICS

The simulation model is not known to be the best technique for optimisation problems. Due to the error in running a finite number of replications and computational speeds, answers obtained via simulation models cannot be shown to be mathematically optimal. PIPER hence utilises the well-known METRIC model (Sherbrooke, 1992) for optimising spare demands that arise from reliability failures. METRIC is an analytical model widely used in the industry for spares optimisation and have shown to provide near-optimal answers. The algorithm consists of two main steps. In the first step, the best location to stock the item is derived via an enumeration of the solution set. The best location is the location that maximises Operational Availability (Ao). The non-dominated set of Ao versus stock quantity (also known as the efficient frontier) is generated.

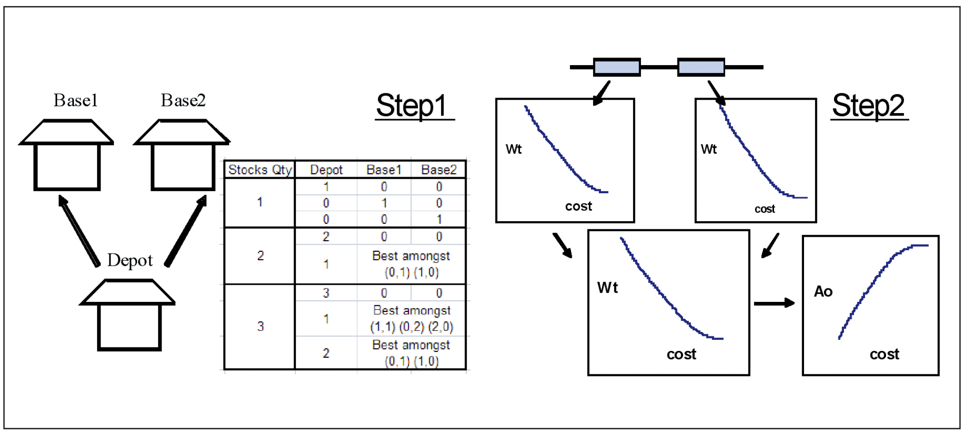



*Figure 5. Optimisation algorithm*

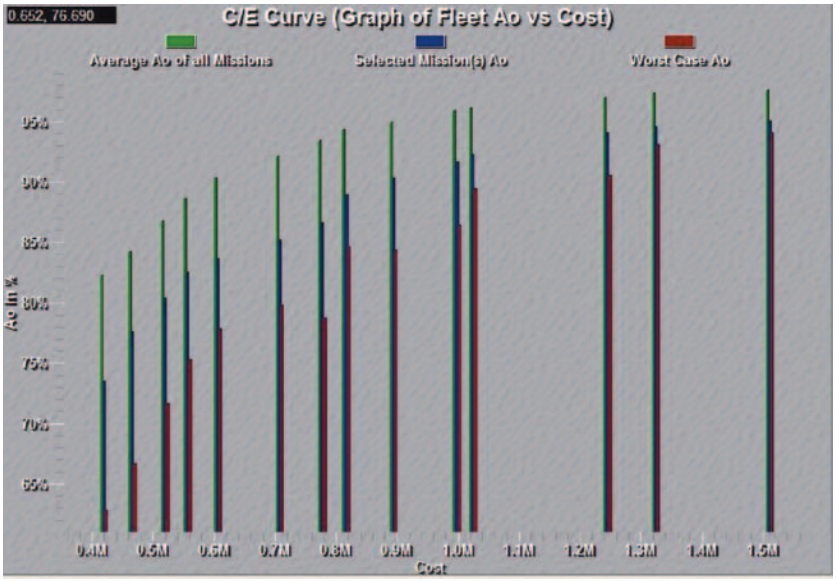



*Figure 6. Graph showing a sample result of spares optimisation*

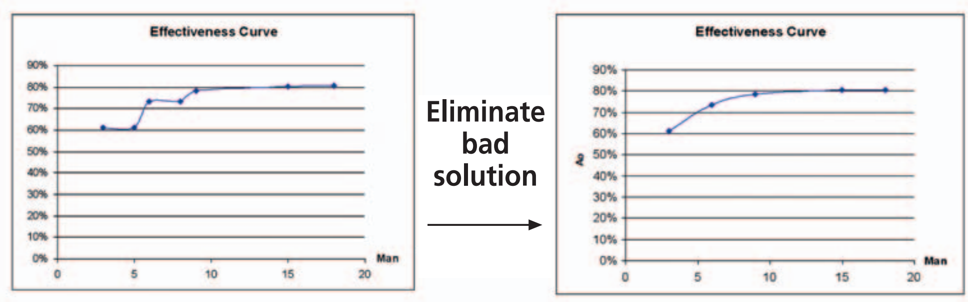
This is carried out for all the LRU types. Combining the various LRU types via marginal analysis (or greedy heuristics) generates the cost-effectiveness curve. A detailed write-up on the algorithm can be found in Sherbrooke, 1992.

The method of marginal analysis optimises spares well but is inadequate for maintenance resources optimisation. This is because a suite of technicians is needed to service a failure and the technicians are shared across LRUs or system types performing LRU removal, replacement as well as repair. The maintenance resources optimisation problem is hence inseparable. It has a non-linear objective function and integer decision variables which make the problem hard. The following is the heuristic adopted to derive the minimum quantity of maintenance resources:

• Run simulation with no technicians. When more than one technician is available to service a job, the job will be assigned to the technician who has worked the most. This serves the purpose of minimising the quantity of technicians. Create technicians only if there is a need, and track the usage of each technician created. The number of technicians created is the quantity that would give zero bottlenecks to the system which is tantamount to 'Infinite spares' and may be huge.

• Backtrack the solution iteratively i.e. run simulation with x% of the total man-hours needed. For example, in iteration 1, set the technician quantity to obtain 30% of the total man-hours that is needed to derive the bottleneck to zero. The technician quantity can be increased from 30% to 40% in iteration 2 and so on.

• Select an operation point on the effectiveness curve. Improve the solution by removing technicians with low usage rate i.e. idle technicians. Re-run to confirm that the Ao is still above the required level.




| Type1Tech | Type2Tech | TotalTech | Ao |
|---|---|---|---|
| 1 | 2 | 3 | 61.4% |
| 2 | 3 | 5 | 61.4% |
| 2 | 4 | 6 | 73.3% |
| 3 | 5 | 8 | 73.3% |
| 3 | 6 | 9 | 78.4% |
| 5 | 10 | 15 | 80.4% |
| 6 | 12 | 18 | 80.7% |

| | Type1Tech | Type2Tech |
|---|---|---|
| LRU1 | 1 | 2 |
| LRU2 | 1 | 2 |
| ... | | |
| LRUn | 1 | 2 |

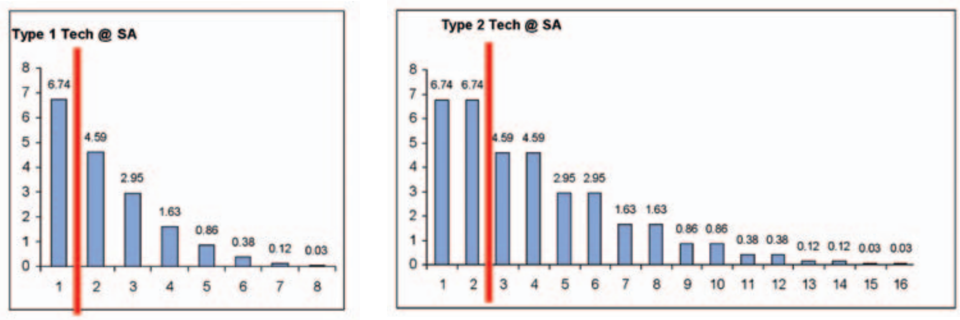



*Figure 7. Results from an Army case study on maintenance resources optimisation*

## SPARE PARTS DEMAND FORECASTING

A large-scale fleet management organisation such as the military often operates equipment at many different sites. However, due to environmental differences, equipment deployed at different operating sites over prolonged periods of time have different observed failure rates. In addition, as most large system houses typically phase in new equipment over many stages, the entire fleet of equipment tends to exhibit different levels of reliability. While it has been a common practice to use a single distribution similar to the study done by Peltz, Colabella, Williams & Boren (2004), this method is clearly not sufficient here. Murthy, Xie & Jiang (2004) described a comprehensive suite of Weibull models. Out of the seven models mentioned therein, Type III (a) model which is a multi-population mixture of related Weibull models looks more appealing because of its proximity to real life scenarios. The authors mentioned that other classes of Weibull models such as Type III (d) Sectional Weibull are a theoretical postulation, and they are unable to offer any physical explanation of the underlying process. Although many methods for multi-population Weibull have been proposed, there is little research done at making a direct comparison between the methods.

The reliability function for a mixture of two Weibull distributions is described below.

$$R(t) = pe^{-(\frac{t}{\alpha_1})^{\beta_1}} + (1-p)e^{-(\frac{t}{\alpha_2})^{\beta_2}} \quad (1)$$

The idea of a mixture of two Weibull distributions was described by Jiang & Kececioglu (1992) and Jiang & Murthy (1995). In this paper, we are following Jiang & Kececioglu's method of parameter estimation, which is summarised as follows:

1. Rearrange the failure time in ascending order. Let $t_i \geq 0, 1 \leq i \leq n$.
2. Compute $x_i$ and $y_i$, $1 \leq i \leq n$, where $x_i = \ln t_i$ and $y_i = \ln\left[-\ln\left(1-\frac{i}{n+1}\right)\right]$ where $\hat{F}_i = \frac{i}{n+1} =$ empirical distribution.
3. Plot $y_i$ vs. $x_i$.
4. Fit a smooth curve to approximate the plotted data.
5. Locate the point T, where the second derivative of the fitted curve is zero.
6. Obtain p from $y_T$ = 1n(-1n $p$).
7. Determine $\beta_1$ and $\beta_2$ from the gradient of the tangent lines of the left and right asymptote of the curve.
8. Determine $\alpha_1$ and $\alpha_2$ by determining the y-axis at 0.632$p$ and $p$ + 0.632(1-$p$) horizontally; intersecting the curve and dropping down, the $\alpha_1$ and $\alpha_2$ can be read from the exponential of the *x*-axis values.

For most practical purposes, we note that the value of $\beta$ ranged from one to five. As spare parts have the property of increasing failure rates in the population, $\beta$ cannot be less than one. When $\beta$ is more than six, it can then be modelled using a mixture of three-parameter Weibull distributions. In addition, all the $\beta$ values within the same mixture of Weibull distributions are constrained to be the same. This is because we assumed the nature of the failure mechanism to be similar. If the $\beta$ values are different, there will be a possibility that the time-to-failure percentile of the stronger population is smaller than the weaker population, which is contrary to intuition. The assumption of the same $\beta$ values is a common assumption made in accelerated life tests which most military equipment undergo. Hence, it is not unreasonable to assume that environmental stress factors affect $\alpha$ values and not $\beta$.

The above parameters estimation algorithm can be generalised for a mixture of n Weibull distributions, with the first four steps being similar. The rest of the steps need to be modified as follows:

- Locate the points, $T_1$, $T_2$,..., $T_{n-1}$, where the second derivative of the fitted curve is zero. Obtain $p_1$ from $y_{T_1}$ = 1n(–1n($p_1$)), $p_2$ from $y_{T_2}$= 1n(-1n($p_2$)),... , and $p_{n-1}$ from $y_{T_{n-1}}$ = 1n(–1n($p_{n-1}$)).
- Determine $\beta$ by taking the average of the two values of slopes of the tangent lines which are drawn at each end of the curve.
- Determine $\alpha_i$ by determining the *y*-axis at the values of expressions (1.1) and (1.2), for $\alpha_1$ and $\alpha_i$ respectively, horizontally; intersecting the curve and dropping down, the $\alpha_i$ can be read from the exponential of the *x*-axis values, where $1 \leq i \leq n$.

The reliability of a mixture of n Weibull distributions is as follows:

$$R(t) = p_1 e^{-(\frac{t}{\alpha_1})^\beta} + p_2 e^{-(\frac{t}{\alpha_2})^\beta} + p_3 e^{-(\frac{t}{\alpha_3})^\beta} + \ldots + (1 - p_1 - p_2 - \ldots - p_{n-1}) e^{-(\frac{t}{\alpha_n})^\beta}$$

When $t = 1, \alpha_1 < \alpha_2, \alpha_1 < \alpha_3, \cdots, \alpha_1 < \alpha_n$,

Divide $\alpha_i$ by $\alpha_1$ throughout,

$$if\ \alpha_i' = \frac{\alpha_i}{\alpha_1} \text{ and } \alpha_1 < \alpha_i$$

$$\Rightarrow \frac{1}{\alpha_i'} = \frac{\alpha_1}{\alpha_i} \approx 0 \text{ for } i = 1,2,\ldots,n$$

Since $e^{-(\frac{1}{\alpha_i'})^\beta} = e^{-(0)^\beta}$, we have

$$\begin{aligned}
R(1) &= p_1 e^{-(\frac{1}{\alpha_1})^\beta} + p_2 e^{-(\frac{1}{\alpha_2})^\beta} + \ldots + (1 - p_1 - p_2 - \ldots - p_{n-1}) e^{-(\frac{1}{\alpha_n})^\beta} \\
&= p_1 e^{-(1)^\beta} + p_2 + \ldots + p_{n-1} + (1 - p_1 - p_2 - \ldots - p_{n-1}) \\
&= 0.368 p_1 + p_2 + \ldots + p_{n-1} + (1 - p_1 - p_2 - \ldots - p_{n-1}) \\
&= 1 - 0.632 p_1 \\
F(1) &= 1 - R(t) \\
&= 0.632 p_1
\end{aligned}$$

When $t = \alpha_1$,

$$\begin{aligned}
R(\alpha_1) &= p_1 e^{-(\frac{1}{\alpha_1})^\beta} + p_2 e^{-(\frac{1}{\alpha_2})^\beta} + \ldots + (1 - p_1 - p_2 - \ldots - p_{n-1}) e^{-(\frac{1}{\alpha_n})^\beta} \\
&= p_1 e^{-(1)^\beta} + p_2 + \ldots + p_{n-1} + (1 - p_1 - p_2 - \ldots - p_{n-1}) \\
&= 0.368 p_1 + p_2 + \ldots + p_{n-1} + (1 - p_1 - p_2 - \ldots - p_{n-1}) \\
&= 1 - 0.632 p_1
\end{aligned}$$

$$F(\alpha_1) = 0.632 p_1 \tag{2}$$

When $t = \alpha_i$
if $\alpha_i < \alpha_k$, where $i < k \le n$

and $e^{-\left(\frac{\alpha_i}{\alpha_l}\right)^\beta} = e^{-(\infty)^\beta} = 0$, where $\le l < i$

$$\begin{aligned}
R(\alpha_i) &= p_1 e^{-(\frac{\alpha_i}{\alpha_1})^\beta} + p_2 e^{-(\frac{\alpha_i}{\alpha_2})^\beta} + \ldots + p_i e^{-(\frac{\alpha_i}{\alpha_i})^\beta} + \ldots + (1 - p_1 - \ldots - p_{n-1}) e^{-(\frac{\alpha_i}{\alpha_n})^\beta} \\
&= 0.368 p_i + p_{i+1} + p_{i+2} + \ldots + p_{n-1} + (1 - p_1 - p_2 - \ldots - p_{n-1}) \\
&= 1 - p_1 - p_2 - \ldots - p_{i-1} - 0.632 p_i \\
F(\alpha_i) &= 1 - R(\alpha_i)
\end{aligned}$$

$$= p_1 + p_2 + \ldots + p_{i-1} + 0.632 p_i \tag{3}$$

Although it is possible to estimate the parameters of a mixture of n Weibull distributions, we focus our discussion on the case of two populations for simplicity. We address the case of complete data i.e. no censoring.

The crux of the estimation problem lies in the approximation of the curve (step four) because the point of inflexion is used to estimate the population mixture, $p_i$. Hence, the curve fitting is an important step. Our main idea is to minimise the absolute deviations of the residuals, which will result in the linear programme below.

Problem CF:

$$\min \sum_{i=1}^{n} |r_i|$$

where $r_i = y_i - g(x)$,

$$y_i = \ln\left(-\ln\left(1-\hat{F}(t_i)\right)\right)$$

$$x_i = \ln(t_i),$$

$\hat{F}(t_i) = \frac{i}{n+1}$, $i =$ order of failure time in ascending order

$g(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4$, $b_i$ is the coefficient of the polynomial

*subject to*

$$\frac{d}{dx}(b_1 + b_2 x + b_3 x^2 + b_4 x^3 + \dots b_n x^{n-1}) \geq 0,\ \forall\ x_i, x_i$$

= $i^{th}$ data point

We penalised the positive deviations (slack) and negative deviations (surplus) equally and constrained the estimated curve, g(x), to be non-decreasing for every data point. Equation (4) is an equivalent formulation of Problem CF.

$$\min \sum_{i=1}^{n} tU_i + (1-t)V_i \text{, where } t = \text{penalty, } 0 < t < 1 \qquad (4)$$

*subject to*

$y_i = g(x_i) + U_i - V_i$, where $g(x)$ is as per above

$U_i >= 0,\ V_i >= 0$

$\hat{F}(t_i)$ is an empirical density function and is non-decreasing, which means $y_i$ is non-decreasing and therefore, *g(x)* needs to be non-decreasing. This is equivalent to fitting a 50% percentile line (the median line) in quantile regression by setting *t* in Equation (2.1) to be 0.5. The $L_1$ line enjoys two main advantages: Firstly, $L_1$ regression contains some robust properties and is superior to $L_2$ (least square line) in *y*-deviations, as mentioned in Rousseeuw & Leroy (1987). Secondly, $L_1$ regression is a linear programme whereas $L_2$ regression is a quadratic programme, which makes $L_1$ more attractive.

For a two-population distribution, we fitted a fourth degree polynomial curve to the plotted points. Though any high order polynomial generally suffices, the main advantage of a fourth degree polynomial is that the second derivative is quadratic which makes the solution of the inflexion point easy.

## CONCLUSION

Due to the increasing need to maintain high combat capabilities while keeping the budget low, it is essential to perform realistic forecasting on the demands of spare parts before using PIPER to improve its accuracy in spares and resources allocation. In our developmental work, we did a comprehensive comparison of the popular methods in the literature. Our test cases have shown that a mixture of multi-population Weibull distributions can be used to represent the demands of spare parts for large-scale fleet scenarios with substantial variability in equipment reliability.

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## BIOGRAPHY

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